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# Modeling and Control of Large Eddies Generated by Maneuvering Self-propelled Bodies in Stratified Fluids

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**Abstract**
Our experiments and theoretical analysis show that when a submerged self-propelled vehicle makes a maneuver in a stratified fluid, e.g., accelerates/decelerates or changes the direction of its motion, this leads to the formation of unusually large horizontal eddies of the size of up to several kilometers and with decay times of several days. It is also shown that for the vertical background shear typical for the upper ocean, the shear itself only partly suppresses the eddy formation and reduces their decay times, which still remain significantly large. Such eddies may have potentially important applications for submarine detection and have not been studied previously.

## Formulation of the problem

Large coherent eddies (monopoles, dipoles and their combinations) are the persistent flow patterns, which are easily formed in stratified/rotating fluids [1,2]. There is a particular interest in planar dipolar (and quasi-dipolar) eddies because they possess a linear (and angular) momentum and can propagate significant distances from the origin. Typical examples include oceanic mushroom-like currents [3] and more complicated self-propagating eddies in the ocean [4-5]. Another example is late stratified wakes behind towed/self-propelled bodies [6-7] including oceanic submarines. Lin and Pao [8] give a review of the work on stratified wakes up to 1979. Since then, a variety of studies have been conducted mostly with towed spheres to investigate the characteristics of the near and late wakes [9-12]. An interesting observation made in these studies is that "pancake" vortex streets can be formed in a late wake at any large but finite initial Froude ($Fr = U/ND$) numbers {12] and the general structure of the late wake does not depend significantly on the shape of the towed body, when the Reynolds ($Re = UD/\nu$) number exceeds a critical value [13] ($U$, $D$ are the body velocity and diameter, $\nu$ is the fluid viscosity and $N$ is the Brunt-Vaisala frequency). Much less is known of stratified wakes produced by self-propelled vehicles. In spite of the technical difficulties in modeling such wakes in the laboratory, some data has been obtained but mostly on the near wake characteristics and only for steady motion [8, 14,15].

In practice, submerged vehicles frequently accelerate/decelerate or change their direction of motion, thus transporting momentum to the fluid (wake). Owing to the fact that the horizontal momentum, transported locally to a stratified fluid, has a strong tendency to generate compact vortex structures, one may expect the appearance of unusually large eddies in the late wakes of maneuvering vehicles. Also Tennekes and Lumley [16] noted this possibility 29 years ago, surprisingly, but there have been no experiments or theories to verify this idea and this topic is addressed in our study.

It is also clear that the background horizontal and vertical shears may significantly modify or destroy such eddies. But because of in the ocean the horizontal shear is typically two to three orders of magnitude smaller than the vertical one, it may be neglected in the first approximation. Therefore, it is of practical interest to investigate the effect of the background vertical shear on the dynamics of late-wake eddies. Previous studies [17,18] include only the case of horizontal (or across-flow) shear. The effect of the vertical shear was not considered previously.

Taking into account technical difficulties, it was decided that the study would not employ a propeller-driven body. Alternatively, the study utilizes a moving force doublet to model correctly the thrust and drag produced by a maneuvering self-propelled body. This concept have been employed before [19], but the method proposed herein is new with regard to obtaining information on the wakes of maneuvering bodies.

## Long-term goal

The long-range goals of our research are to improve a basic understanding and, eventually, develop a predictive capability of the initiation, temporal evolution and decay of large eddies which are formed in late stratified wakes of maneuvering oceanic submarines. This problem is closely related to the general problem of compact eddies generated in stratified/rotating fluids when linear or angular momentum is transported locally to the fluid [2] and is far from complete understanding.

## Research objectives

The main objectives of our study may be formulated as follows:

(i) to model in laboratory experiments late wake eddies behind a maneuvering (non-steady motion) self-propelled vehicle;

(ii) to study how the formation time, lifetime, size and other properties of these eddies depend on the system parameters;

(iii)to reproduce and to study basic compact eddies with linear and angular momentum;

(iv)to investigate the influence of the background shear on the formation and evolution of large eddies;

(v) to develop physical model.

## Work completed

During the grant period (02/1999 - 01/2001) the following work was completed:

1. The combination of a moving force doublet and a momentum source was used in experiments to model the late stratified flow behind maneuvering (non-steady motion) self-propelled vehicle. Experiments were conducted with co-flow, counter-flow and cross-flow configurations to model the effects of rectilinear acceleration/deceleration and change of the direction of motion of a self-propelled vehicle. It is shown that at least two possibilities may be realized in a late flow and two different flow patterns may be observed in the late wakes: (i) a system of large dipolar patches with finite momentum or (ii) one unusually large dipole. The conditions under which large eddies appear in the flow as well as their characteristics (formation time, lifetime, size) are determined in terms of the critical system parameters.
2. Experiments were performed on the dynamics of basic self-propagating eddies generated in a volume of fluid when both linear (P) and angular (M) momentum are applied locally to a fluid. Using the method proposed, it is possible to generate a whole family of isolated (net vorticity is equal to zero) eddies with different values of the non-dimensional parameter $\varepsilon$, which is proportional to the ratio of linear to angular momentum ($\varepsilon$ = RP/M, R is the eddy size).
3. Experiments were conducted with the purpose of clarifying the conditions under which the large eddies are influenced by the background shear. The flow regime diagram is derived and conditions under which the formation of dipolar eddies is possible are found and explained. The estimates for the dipole lifetime are given.

## Main results

A brief summary of the main results and some examples of characteristic flow patterns are given below. For more detail we refer the reader to our publications, which are given at the end of this report

### 1. Momentum self-propelled wakes and large eddies

The experiments were performed using a long tank (10m x 0.5m x 0.5m), filled with linearly stratified (by salt) water of $N \approx 1s^{-1}$. The tank is equipped with a computer controlled driving mechanism that translates a small platform with a variable horizontal speed *U(t)*. Steady and unsteady rectilinear motions of a self-propelled body were reproduced using a model (Fig.1) attached to the platform. The combination of a solid body and a jet (attached to the body) was used to model self-propelled wakes. Note that the carriage in this

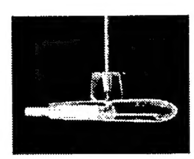


*Fig. 1.*

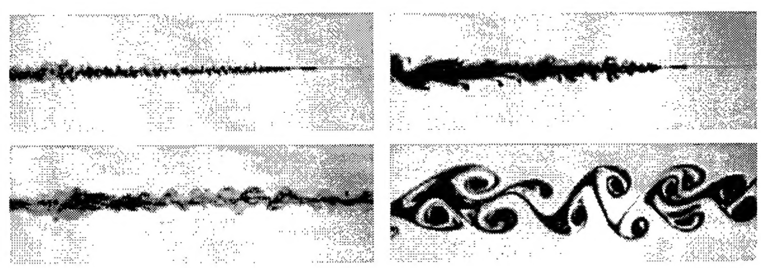
method plays only the role of a guide for the model, because it moves exactly with the same velocity $U(t)$ that a free model would move with a given thrust. The main idea here is as follows. The balance of momentum for a self-propelled body of mass $M$ moving with horizontal velocity $U$ may be written as $(1+k)\ M\ dU/dt = F_1 - F_2$ ,where $F_1$ is the thrust reaction on the body, $F_2$ is the drag force and $k$ is the virtual mass coefficient. For steady motion $F_1$-$F_2$ = 0, the net momentum applied to the fluid is zero and the action of the body on the fluid is equivalent to the action of a force doublet of intensity $Q = 2\delta F_1$, moving with the speed $U$, where $\delta$ is the effective distance between these two forces. When the body, e.g., accelerates, $F_1 - F_2 = F > 0$, the action of the body on the fluid can be thought of as equivalent to the combined action of a force doublet of intensity $Q$ plus the action of a force $F$. As a result, a self-propelled wake acquires a momentum $\rho I = F\ T_0$ ($T_0$ is the duration of the acceleration, $\rho$ is the fluid density). In this method, the drag is produced by the moving body and can be estimated as: $F_2 / \rho = J_2 \approx C_d S U^2 / 2$ ($C_d \approx 0.6$ is the drag coefficient, $S = \pi D^2 / 4$), while a jet models the thrust with the intensity $F_1 / \rho = J_1 \approx q(q - sU) / s$ ($d$ is the nozzle diameter, $s = \pi d^2 / 4$. By changing small volume flux $q$ from the nozzle, $J_1$ can be changed and a combination of a force doublet and a force can be reproduced. Using these parameterizations, the balance of momentum becomes

$$(1+k)SLdU / dt = q(q - sU) / s - C_d S U^2 / 2, \tag{1}$$

and this gives the relation between $q$ and $U(t)$, which must be satisfied in the experiment. If $q$ does not depend on time, the solution for $U$ is

$$(U - U_1)/(U - U_2) = [(U_0 - U_1)/(U_0 - U_2)] \exp[-(b/a)(U_1 - U_2)t] \tag{2}$$

where $a = (1+k)SL$, $b = C_d S/2$, $U_{1,2} \propto q$ are real roots of the equation $bU^2 + qU - q^2/s = 0$, and $U(t) = U_0$ at $t = 0$.



(a) (b)

***Fig 2.*** *Zero-momentum (a) and momentum (b) self-propelled wakes in a stratified fluid. Combination of solid body (moving from left to right and visible in the top pictures) and jet (attached to the body and acting in opposite direction) is used in this example to model self-propelled wakes (see Fig. 1). Near (top) and late (bottom) wake patterns are shown.*

Typical examples of flow patterns observed at different configurations are shown in Figs. 2-5. When thrust is equal to drag, the self-propelled body moves steadily. The wake has zero net momentum and there are no noticeable eddies in a late flow (Fig. 2a). When the thrust exceeds the drag, the self-propelled body accelerates and the wake acquires momentum in the direction of thrust. At large times the

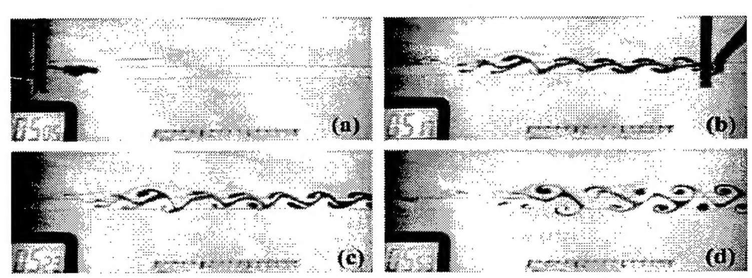



***Fig. 3.*** *Formation of a system of large eddies in a late flow induced by a decelerating self-propelled body which moves from left to right and applies momentum to the fluid in the same direction.*

momentum wake becomes unstable and a system of large eddies, similar to a vortex street, arises in a late flow (Fig. 2b). When a self-propelled body decelerates (Fig. 3), a system of eddies again arises in a late flow. The main difference between accelerating and decelerating motions is the "direction" of the resulting vortex street, which coincides with the direction of momentum applied to the fluid. Important result here is that in both cases the initial size $L_0$ of the late eddies in a vortex street is not related to the size of self-propelled body (as in the case of towed cylinder or sphere) but rather depends on $J = F/\rho$ and U as $L_0 \approx 6J^{1/2}/U$. With time the size of eddies increases partly because of horizontal entrainment, but mostly because of eddies interaction and the resulting periodic doubling of the eddy scale when two nearest eddies of the same sense of rotation form one bigger eddy (Fig. 4).

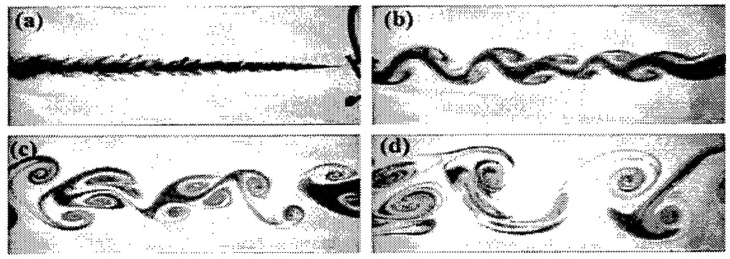



*Fig. 4. Formation of eddy system and doubling of eddy scale in a stratified flow induced by a moving (from left to right) and continuously acting in the same direction momentum source.*

Using the method proposed, more complicated cases of non-steady motion may be reproduced. In one of the experiments the model was initially still and $q = 0$. Then, at $t = 0$, the jet was initiated with $q = q_1$ and the model was moved with the velocity $U(t)$ as calculated from (2). After the model reached its terminal speed $U \approx U_1(q_1)$, the model was moved with this

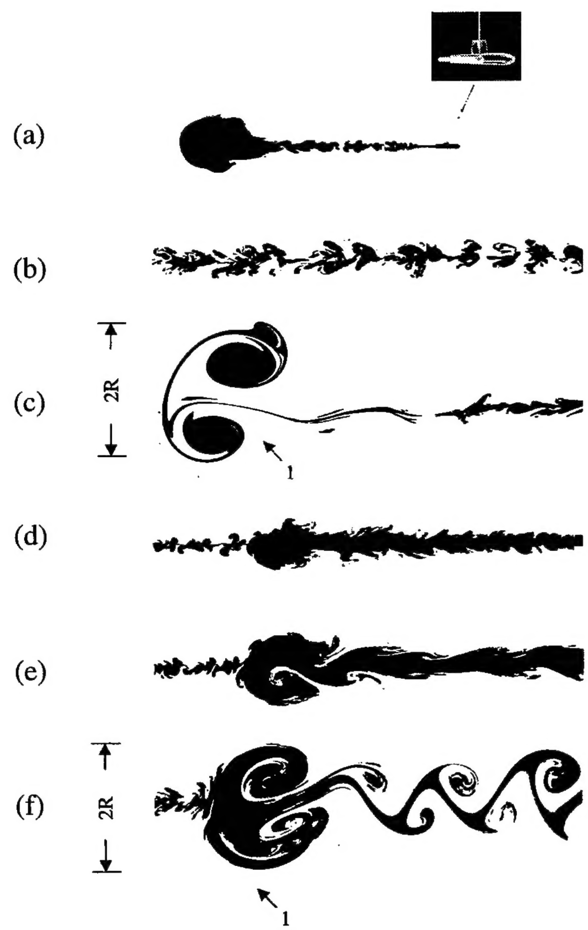



*Fig. 5. Formation of unusually large eddies when a model submarine: (a) starts moving from left to right, (b) moves with constant speed, (d) accelerates again. With time large eddy (1) is formed in the late starting flow (c). Similar structure (1) is formed in the late momentum wake (f) as a result of the model acceleration. Zero-momentum wake (b) rapidly decays, while momentum wakes (c, f) remain active for a long time.*

constant speed during some time interval and then the value of $q$ was increased to a new value $q = q_2$. Simultaneously the model velocity $U(t)$ was increased in accordance with (2), and after some time it reached its new terminal value $U \approx U_2(q_2)$.

Typical results of such experiments are shown in a sequence of photographs in Fig. 5. Initially (Fig. 5a), when the submarine starts moving and accelerates, the thrust exceeds the drag and the momentum is imparted to the fluid. As a result, a large dipolar eddy develops with time in this starting flow (Fig. 5c). After acceleration, the body moves with constant speed and produces a zero-momentum wake (Fig. 5b), because the thrust now is balanced by the drag. The zero-momentum wake decays rapidly without the formation of any large eddies. After traveling some distance with constant speed, the submarine accelerates again and generates a momentum wake (Fig. 5d). With time, a second large dipolar eddy forms in the late wake (Figs. 5e,f). As a result of shear instability (or some other mechanism), this large eddy is accompanied by a regular system of less intense eddies, similar to that shown in Fig. 2, and this system of eddies moves slowly in the same direction as the large one (from the body).

Approximate conditions at which the late flow patterns change from a system of eddies to one large eddy may be formulated using a dimensionless parameter $A = U^2\Delta t^{3/2}/I^{1/2}$. At $A > A_{cr}$ a system of eddies may be expected and at $A < A_{cr}$ only one large eddy.

The thickness $H$ and horizontal size $2R$ of large eddies, generated during such non-steady body motion, were measured at different times and example is shown in non-dimensional form in Fig. 6. Time origin $t = 0$ in this graph coincides with the instances where $q$ and $U$ were changed. The excess of momentum $I$ transported to the fluid in the experiment can be estimated from (1). This gives $I \approx (1+k)SL\Delta U$ and $H$ and $2R$ of the resulting dipolar eddies can be estimated using

$$H = \gamma(I/N)^{1/4}, \qquad 2R = \alpha[12/\pi\gamma\alpha^2(1+k)]^{1/3} N^{1/12} I^{1/4} t^{1/3} \tag{3}$$

($\gamma = 1.5$, $\alpha = 0.5$, $t$ is the time). For comparison a solid line in Fig. 6, as calculated using (3), shows the estimated values of $2R/H$.

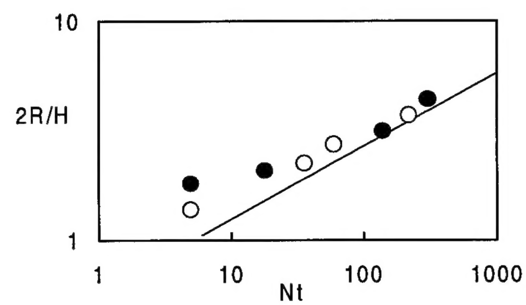



***Fig. 6.*** *Typical non-dimensional size, 2R/H, of large eddies generated by accelerating body for different non-dimensional times, Nt, after the beginning of acceleration. Symbols – measured values, solid line – theory, (●) – first (starting) eddy, (○) – second eddy.*

For the cases of rectilinear motion with acceleration/ deceleration, considered above, the momentum is transported to the wake in the directions parallel to the direction of body motion. When a self-propelled body changes its direction of motion, momentum is transported to the fluid mostly in the direction normal to the body pass. In the experiment (Fig. 7) the body moves horizontally with constant velocity from right to left (a) and the

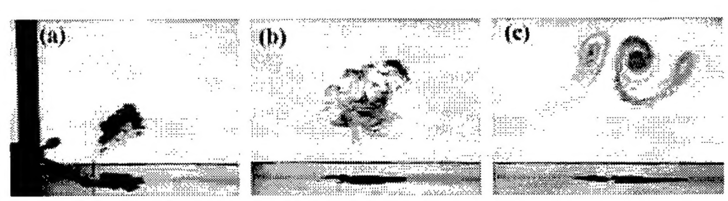



***Fig. 7.*** *Formation of large eddy in a late flow when a body moves from left to right and applies a momentum in the direction normal to the body pass. For comparison, side view is also given at the bottom.*

momentum source acts for a short time interval in the horizontal direction which is normal to the body pass. When $A < A_{cr}$, only one large eddy, moving normal to the body pass, is formed in the late flow (Fig. 7b,c).

2. _Self-propagating quasi-monopoles_

In general, in addition to a linear momentum, maneuvering submarine may transport to the fluid (wake) also an angular momentum. Thus, one may expect that large eddies with both linear and angular moments may be formed in a late flow. Such eddies are called sometime as self-propagating quasi-monopoles. In contrast to dipolar eddies, self-propagating quasi-monopoles are not well studied and only recently they were analyzed theoretically [4]. The main difficulty in experimental modeling of such eddies is controlling the forcing. To generate dipolar eddies only linear momentum should be transmitted to the fluid and this can be done using jets and the intensity of forcing can be calculated. To generate self-propagating quasi-monopoles one needs to produce and estimate both linear and angular momenta. This, however, is not an easy task and, for example, the use of maneuvering model submarines for these purposes looks very problematic because of technical difficulties. Taking all these into account we decided to use a much simpler experimental method to generate eddies with controllable angular and linear momenta. The main purpose of the experiments was to verify the theoretical predictions [4], namely to demonstrate that such eddies exist, that they are stable under curtain conditions and their basic characteristics may be described using the theory.

The experiments were conducted using a square tank 75x75 cm. As a first step, a layer of homogeneous water (depth 10 cm) was used. To make the flow quasi-two-dimensional, the tank was mounted on a turntable rotating in an anti-clockwise direction at a constant rate $\Omega_0$. The eddies with controllable angular and linear moments were generated using a stirring technique. For this purpose a heavy bottomless cylinder (diameter $2R$ = 10 cm) with thin walls was introduced into the tank. The fluid inside the cylinder was stirred cyclonicaly with an angular velocity $\Omega$. Then the cylinder was pushed forward with a horizontal velocity $U_0$ and at the same time the cylinder was smoothly removed vertically from the water. The flow induced by this method has zero net vorticity and presents a combination of a monopole with angular momentum $M$ ($\propto \rho R^4 \Omega$) and a dipole with linear momentum $P$ ($\propto \rho R^2 U_0$) per unit depth. Changing the non-dimensional ratio $2RP/M \propto 2U_0/R\Omega = \varepsilon$, it is possible to reproduce a whole family of eddies (monopole, $\varepsilon = 0$, quasi-monopole, $\varepsilon \approx 0.1$-$0.5$, quasi-dipole, $\varepsilon \approx 0.5$-$2$, and dipole, $\varepsilon >> 1$). An example showing the formation and propagation of stable quasi-monopolar eddy with relatively large angular momentum is given in Fig. 8. As can be seen, the eddy is stable and steadily propagates for a large distance from the origin with velocity predicted by the theory (Fig.9).

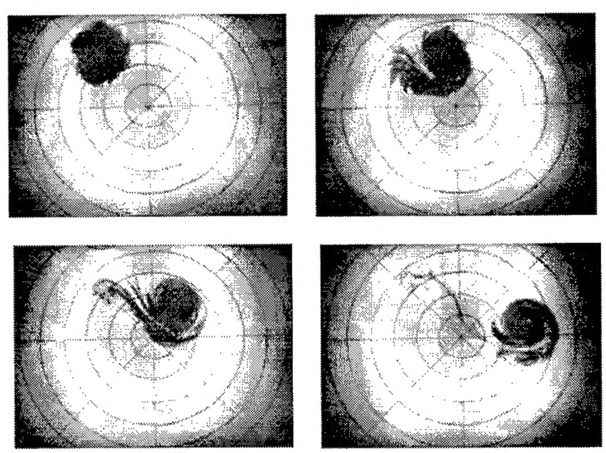


*Fig. 8. Formation and propagation of isolated (zero net vorticity) quasi-monopole. Experimental parameters: $\Omega \approx 2\ s^{-1}$, $U_0 \approx 0.5$ $cm\ s^{-1}$, $\varepsilon \approx 0.12$.*

To obtain the velocity and vorticity distributions in the flow, the fluid was seeded with small tracer particles and a particle image

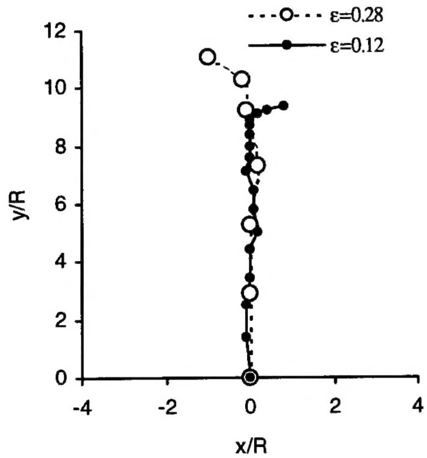



*Fig 9. Trajectories of two eddies (with different ε) from the moment t = 0. For comparison with similar data calculated in [4], non-dimensional coordinates (x/R, y/R) and non-dimensional time (Ωt) are used. The non-dimensional time interval between two nearest points (for the same eddy) is equal to 20*

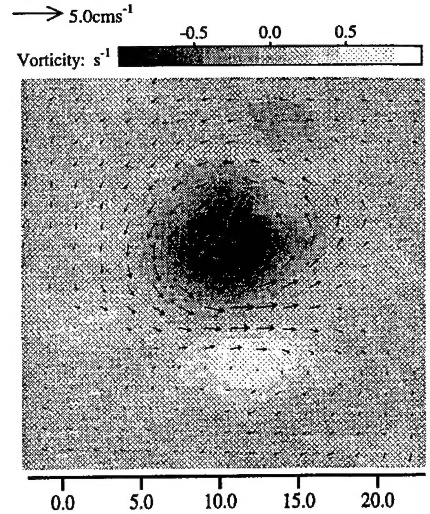



*Fig. 10. Instantaneous velocity field for the steady moving quasi-monopole is shown by arrows while the corresponding vorticity distribution is given by gray shadows. The particle tracking was made in the same experiment as shown in Fig. 8.*

velocimetry procedure was used. Typical results of measurements are shown in Fig. 10. Using these data, vorticity contours, eddy trajectories, etc, were reconstructed and compared with the theory.

3. Dipolar eddies in a shear flow

In experiments we used a water channel, similar to "Odell-Kovasznay" recirculating water tank [20], to produce controllable vertical shear. Linearly stratified (by salt) water with a constant buoyancy frequency N ($= 2\pi/T_N$) is driven around the channel by a viscous drag produced by two vertical stacks of thin Plexiglas disks. The upper and lower parts of each stack rotate in opposite directions and water in the upper and bottom layers moves horizontally with the velocity u(z) (z is the vertical coordinate) in opposite directions. The resulting shear $T_S$ = du/dz was almost constant in the central part of the test section, with approximately zero mean velocity at the mid-level.

Dipolar eddies with controllable intensity (linear momentum $I = JT_0$) were generated by injecting, during a time interval $T_0$, a small amount of dyed and neutrally buoyant fluid (volume flux q) from a thin horizontal nozzle (diameter d), which was located at the mid-level (zero mean velocity). The source of strong turbulent motion in this case is equivalent to a concentrated momentum source of intensity $J$. The kinematic momentum flux $J = q^2/s$ ($s= \pi d^2/4$) and the net momentum during the time interval $T_0$ is $I = JT_0$ [21].

In our experiments with shear, two qualitatively different flow regimes were identified. The first regime corresponds to the case when the dipoles were formed, while the second one represents the case when dipoles do not form. The first regime was observed at relatively weak values of shear ($T_S \geq T_0, T_N$) and an example is shown in Fig. 11. As can be seen, the flow

development is qualitatively similar to the case of zero shear [22] with the resulting planar dipolar eddy. The main difference here is that the initial turbulent blob (Fig. 11a) is not so compact as in the case of zero shear and somewhat dispersed in the along flow direction. Nevertheless, the tendency to collect vorticity into a compact vortical region prevails at this stage (Fig. 11a,b) over the effect of dispersion by shear. With time a planar dipolar eddy is formed (Fig. 11c). The lifetime of the formed dipole is less than in the case of zero shear because at larger times (Fig. 11d) the shear flow effectively washes away the upper and lower parts of the formed dipole. Thus, the main influence of the background shear in the considered regime is that the resulting dipole becomes noticeably dispersed in the along flow direction, and its lifetime is reduced compared to the case of zero shear.

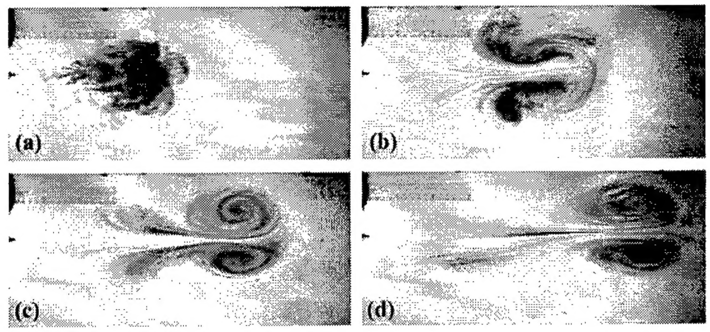



***Fig. 11.*** *Formation of dipolar eddy in a stratified shear flow. Top view. The background shear flow moves from left to right above the dipole mid-plane and from right to left below it. Experimental parameters: $T_S = 4$ s, $T_0 = 4.4$ s, $T_N = 1.6$ s ($Re = J^{1/2}/\nu = 990$). Time t in seconds from the beginning of the experiment: t = 5 (a), 19 (b), 39 (c), 79 (d).*

The second regime was observed at relatively strong shears ($T_S < T_0, T_N$). In this case, Fig. 12, although the formation of the vorticity front begins in the starting flow (Fig. 12a), the dipole does not form (Fig. 12b,c). This is due to the fact that the dispersion by the shear dominates over the accumulation of the flow into a compact vortex region. As a result, at larger times (Fig. 12b,c), when the stratification becomes important, the dipole doesn't form because of the strong dispersion of the initial 3D turbulent spot in the horizontal along flow direction.

From these two examples it is clear that the vertical shear may significantly change the flow regime. It either prevents the formation of the dipolar eddy or the eddy is formed and its lifetime reduced compared to the case of zero shear. The critical conditions when the dipolar eddies either form or not, are given in the flow regime diagram in Fig. 13. A model was developed to explain the results of experiments and theoretical criterion is also shown by dashed and solid lines in Fig. 13. As can be seen, the proposed criterion satisfactorily separates two data point sets, where the dipole does and does not form.

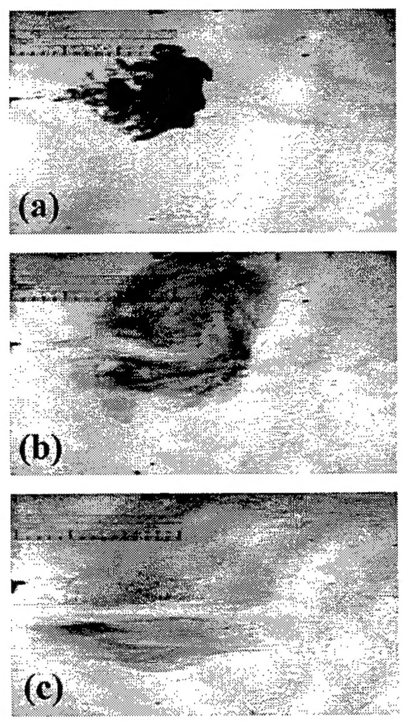



***Fig. 12.*** *The same as in Fig. 11, but at relatively strong shear. Experimental parameters: $T_S = 2$ s, $T_0 = 2.4$ s, $T_N = 4$ s ($Re = 1060$); t = 3 (a), 22 (b), 53 s (c).*

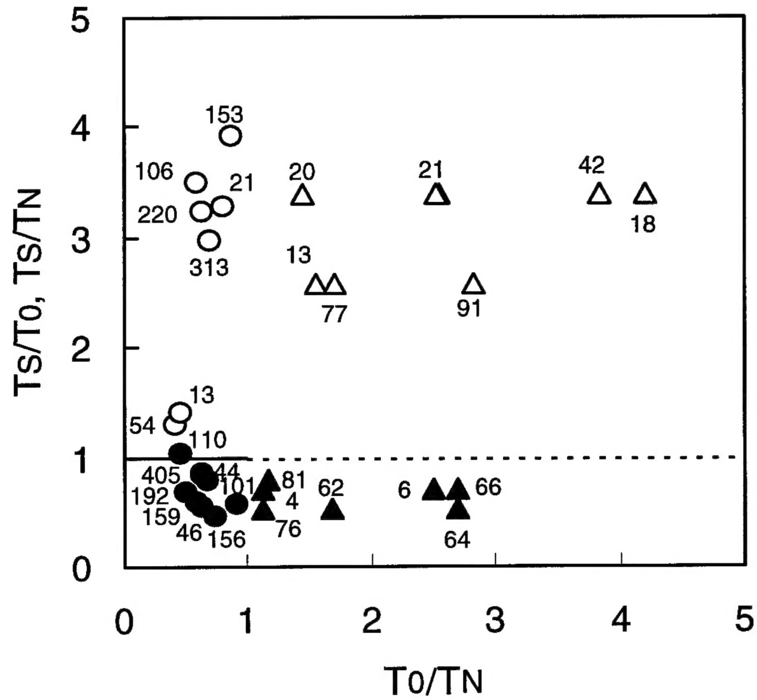



*Fig. 13. Flow regime diagram for the conditions of dipole formation. Symbols - results of experiments. Open symbols - cases when dipoles were formed; solid symbols - cases when dipoles were not formed; circles - results of the experiments when $T_0/T_N < 1$ ($T_S/T_0$ should be used as a vertical coordinate); triangles - results of the experiments when $T_0/T_N \geq 1$ ($T_S/T_N$ should be used as a vertical coordinate). Solid ($T_0/T_N < 1$) and dashed ($T_0/T_N \geq 1$) lines show the transitional criterion between two flow regimes. To show that the flow regime dos not depend on the source intensity, the values of J (in $cm^4s^{-2}$) are given near symbols.*

To estimate the lifetime $T_L$ of the formed dipole, the flow kinematics was considered and formula for $T_L$ derived. Comparison with the measurements shows satisfactory agreement.

**Conclusions**

When a submarine makes a maneuver, e.g., accelerates/decelerates or changes the direction of its motion, significant momentum is transported to the surrounding fluid. Our study show that in a stratified fluid this may lead to the formation of unusually large eddies, much larger and different from those produced in the late wake during steady motion. Comparison with the theory shows satisfactory agreement. Estimates also show that when an oceanic submarine changes its velocity by 10 % or its direction of motion by 5 degrees, large eddies of the size of 1-2 km and with decay times of several days may be expected.

Experiments on the dynamics of basic self-propagating eddies with both linear and angular momentum were performed. Using the method proposed, it is possible to generate a whole family of isolated (net vorticity is equal to zero) eddies with different values of the non-dimensional parameter $\varepsilon$, which is proportional to the ratio of linear to angular momentum. The most interesting case related to oceanographic applications occurs when $\varepsilon$ is small but finite. Satisfactory agreement between the results of experiments and theory proposed in [4] is found.

The influence of the background vertical shear on the dipole formation and evolution was studied. The flow regime diagram is determined and conditions under which the formation of dipolar eddies is possible are found and explained. The estimates show that for the vertical background shear typical for the upper ocean, the shear itself only partly suppresses the eddy formation and reduces to some extent their decay times $T_L$. For example, our model gives for oceanic mushroom-like currents the estimate $T_L \approx 5\text{x}10^5$ s (which is in agreement with satellite observations [3]) and predicts for eddies generated by maneuvering submarines $T_L \approx 7\text{x}10^4$ s

(which is less then predictions for the case when the shear is absent but still significantly large to be important in applications).

Direct naval applications for the results obtained may include such actual Navy tasks as (i) detection and monitoring of large eddies as a characteristic signature of the late submarines wakes and (ii) control of large eddies generation with the purpose of minimizing submarine wake signature.

To the best of our knowledge, this study is the first attempt to improve a basic understanding and, eventually, develop a predictive capability of the initiation, temporal evolution and decay of large eddies which are formed in late stratified wakes of maneuvering oceanic submarines. Prior to the present research the literature on this problem was practically absent. We submit that we only touched the "tip of the iceberg"and much more should be done.

**References**


1. E.J. Hopfinger and G.J.F. van Heijst, "Vortices in rotating fluids", *Ann. Rev. Fluid Mech.,* **25**: 241-289 (1993).
2. S.I. Voropayev and Ya. D Afanasyev *Vortex Structures in a Stratified Fluid.* Chapman & Hall, London (1994).
3. K.N. Fedorov and A.I. Ginzburg, *"The Near-surface Layer of the Ocean",* VSP (1992).
4. M.E. Stern and T. Radko, "The self-propagating quasi-monopolar vortex", *J. Phys. Oceanogr.*, **28**, 22 (1998).
5. S.I. Voropayev, G.B. McEachern, D.L. Boyer and H.J.S. Fernando, "Experiment on the self-propagating quasi-monopolar vortex", *J. Phys. Oceanogr.,* **29**(10), 2741 (1999).
6. G. R. Spedding, "The evolution of initially turbulent bluff-body wakes at high Froude number", *J. Fluid Mech.*, **337**, 283 (1997).
7. S.I. Voropayev, G.B. McEachern, H.J.S. Fernando and D.L. Boyer, "Large vortex structures behind a maneuvering body in stratified fluids", *Phys. Fluids*, **11**(6), 1682 (1999).
8. J. T. Lin and Y-H. Pao, "Wakes in stratified fluids," *Ann. Rev. Fluid Mech.*, 11, 317 (1979).
9. Q. Lin, W. Lindberg, D. L. Boyer and H. J. S. Fernando, "Stratified flow past a sphere," *J. Fluid Mech.* 240, 315 (1992).
10. J. M.Chomaz, P. Bonneton and E. L. Hopfinger, "The structure of the near wake of a sphere moving in a stratified fluid," *J. Fluid Mech.* 254, 1 (1993).
11. G. R. Spedding, F. K. Browand and A. M. Fincham, "Turbulence, similarity scaling, and vortex geometry in the wake of a sphere in a stable-stratified fluid", *J. Fluid Mech.* 314, 53 (1996).
12. G. R. Spedding, "The evolution of initially turbulent bluff-body wakes at high Froud number," *J. Fluid Mech.* 337, 283 (1997).
13. S. I. Voropayev and I. A. Filippov, "Vortical track behind three-dimensional body moving in stratified fluid," *Morskoy Gidrofiz. Zhurnal* 6, 62 (1985) (in Russian).
14. A. H. Schooley and R. W. Stewart, "Experiments with a self-propelled body submerged in a fluid with vertical density gradient," *J. Fluid Mech.* 15, 83 (1963).
15. H. E. Gilreath and A. Brandt, "Experiments on the generation of internal waves in a stratified fluid," *AIAA J.* 23, 693 (1985).
16. H. Tennekes and J. L. Lumley, *A First Course of Turbulence* (The MIT Press, 1972).
17. R.R. Trieling, J.M.A. van Wesenbeeck and G.J.F. van Heijst, "Dipolar vortices in a strain flow", *Phys. Fluids*, **10**(1), 144 (1998).
18. D.P. Delisi, R.E. Robins and R.D. Lucas, "Initial laboratory observations of the evolution of a vortex pair in a stratified shear flow", *Phys. Fluids*, **3**(11), 2489 (1991).
19. E. Naudascher, "Flow in the wake of self-propelled bodies and related sources of turbulence,". *J. Fluid Mech.* 22, 625 (1965).
20. G.M. Odell and L.S.G. Kovasznay, "A new type of water channel with density stratification", *J. Fluid Mech.*, **50**, 535 (1971).

21. H. Schlichting, "*Boundary-Layer Theory*", McGraw-Hill (1979).
22. S.I. Voropayev, Ya.D. Afanasyev and I.A. Filippov, "Horizontal jets and vortex dipoles in a stratified fluid". *J. Fluid Mech,*, **227**, 543, 1991).

## Publications

1. Voropayev, S.I., McEachern, G.B., Fernando, H.J.S. and Boyer, D.L. 1999. Large vortex structures behind a maneuvering body in stratified fluids. *Phys. Fluids*, 11(6), 1682-1684.
2. Voropayev, S.I., McEachern, G.B., Boyer, D.L. and Fernando, H.J.S. 1999. Experiment on the self-propagating quasi-monopolar vortex. *J. Phys. Oceanogr.*, 29(10), 2741-2751.
3. Voropayev, S.I. 2000. Jets and vortex structure formation and interactions in stratified and rotating fluids. *Turbulent Mixing in Geophysical Flows* (Eds. P.F. Linden and J.F. Redondo). CIMNE, Barcelona, 225-247.
4. Voropayev, S.I., Smirnov, S.A. and Brandt, A. 2000. Dipolar eddies in a stratified shear flow. *Stratified Flows, V. I* (Ed. G. Lawrence, R. Pieters, N. Yonemitsu), UBC, Vancouver, Canada, 143-148.
5. Voropayev, S.I., Smirnov, S.A., Filippov, I.A. and Boyer D.L. 2000. Large eddies and vortex streets behind moving jets in a stratified fluid. *Stratified Flows, V. I* (Ed. G. Lawrence, R. Pieters, N. Yonemitsu), UBC, Vancouver, Canada, 149-154.
6. Voropayev, S.I., Smirnov, S.A. and Brandt, A. 2001. Planar dipolar eddies in a stratified flow with vertical shear. *Phys. Fluids*, under revision